University of California, Santa Barbara

Department of Electrical and Computer Engineering

ECE 152A – Digital Design Principles

Homework #6 – Solution

Problem #1.

In this problem you are to design a controller for a vending machine that dispenses 37¢ stamps. The machine accepts only quarters and a maximum of 3 quarters can be inserted (at which time the machine will deliver stamps, change and reset itself to the initial state).

Inputs to the machine are Q (quarter) indicating a quarter has been deposited, S (stamp) indicating that the user wants stamps delivered and C (cancel) indicating that the user wants to cancel the transaction and have his money returned.

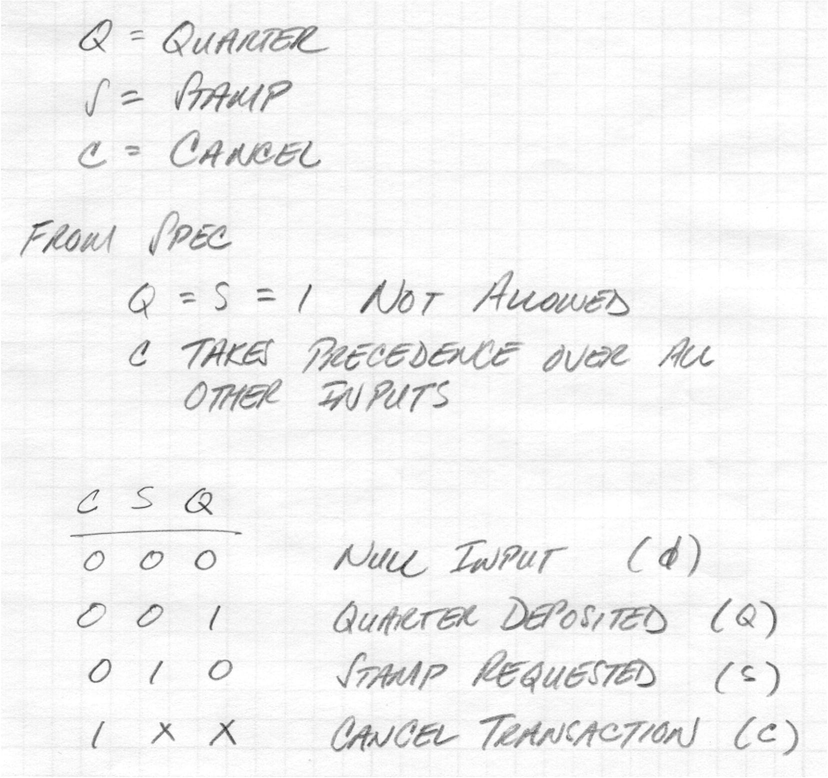
If the S (stamp) input is detected and 50¢ has been deposited, the machine should deliver a single 37¢ stamp (and the change described below). If the S (stamp) is detected before 50¢ has been deposited, an error (ERR) signal is generated. A two bit output from the controller indicates how many stamps should be delivered (STMP1, STMP0). After delivering the stamps, the machine should return to the initial state.

The machine returns only nickels for change; any additional (single penny) change is returned in the form of 1¢ stamps. A two bit output (NCK1, NCK0) indicates the number of nickels to be returned. A two bit output (PNY1, PNY0) indicates how many 1¢ stamps to deliver.

If at any time the C (cancel) signal is detected a coin return (CR) signal is generated causing the contents of the coin box to be emptied and the machine to be reset to the initial state. The cancel/coin return operation has the highest priority and should be executed regardless of the value of S and Q. Also, assume that S (stamp) and Q (quarter) can never be exerted simultaneously.

Design the controller as a Moore machine and include the following:

1. A listing of all valid input combinations (based on the spec above).



Q = QUARTER
S = STAMP
C = CANCEL

FROM SPEC

Q = S = 1 NOT ALLOWED
C TAKES PRECEDENCE OVER ALL OTHER INPUTS

C	S	Q	
0	0	0	NULL INPUT (ϕ)
0	0	1	QUARTER DEPOSITED (Q)
0	1	0	STAMP REQUESTED (S)
1	X	X	CANCEL TRANSACTION (C)

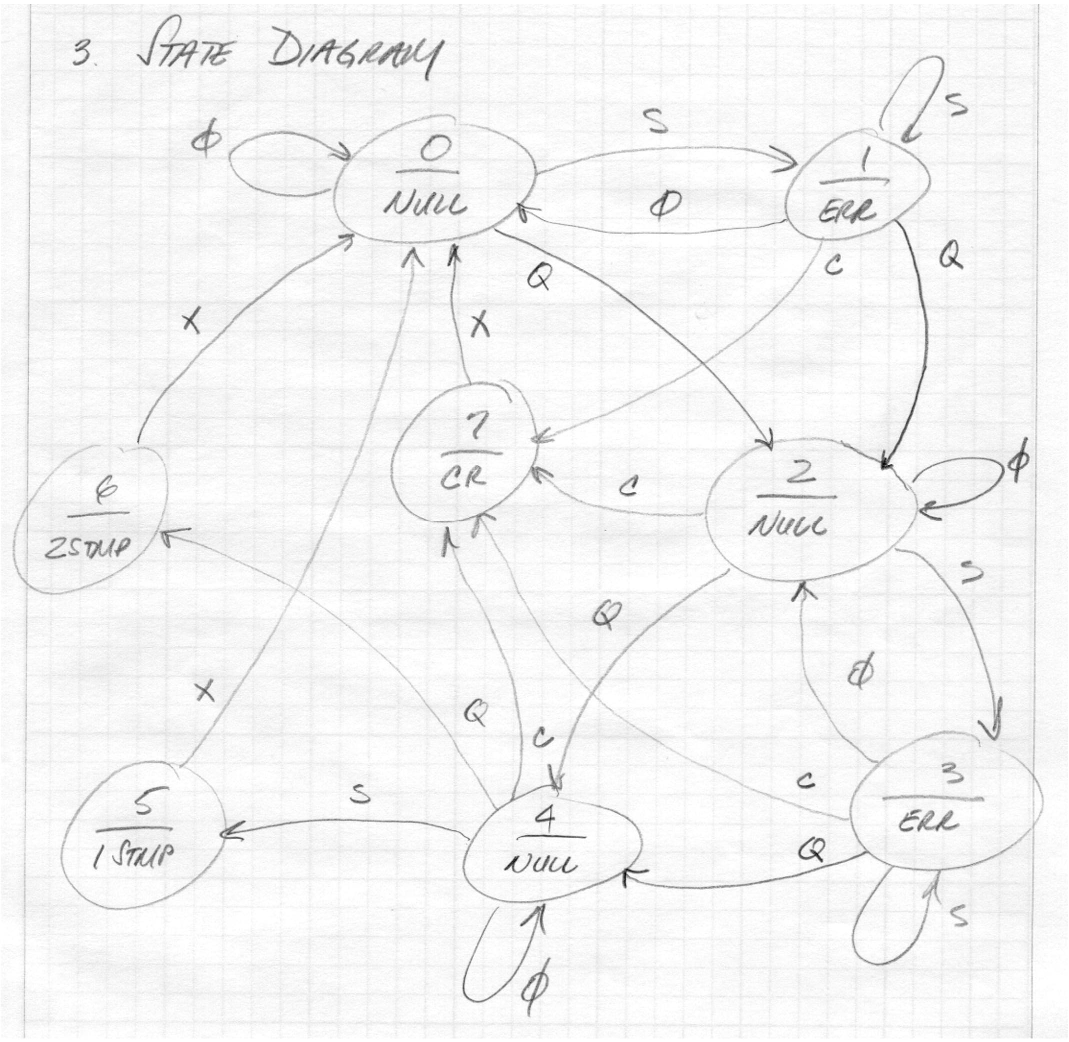
2. A listing of all valid output combinations (based on the spec above).

CR = COIN RETURN
ERR = ERROR
STMP1, STMP0 = # OF STAMPS (0, 1 OR 2)
NCK1, NCK0 = # OF NICKELS IN CHANGE (0 OR 2)
PNY1, PNY0 = # OF 1¢ STAMPS (0, 1 OR 3)

VALID COMBINATIONS

<table>
<tr><th>CR</th><th>ERR</th><th>STMP1</th><th>STMP0</th><th>NCK1</th><th>NCK0</th><th>PNY1</th><th>PNY0</th></tr>
<tr><td colspan="8">NULL OUTPUT:</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td colspan="8">ERROR OUTPUT:</td></tr>
<tr><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td colspan="8">DELIVER 1 STAMP, 2 NICKELS, 3 1¢ STAMPS:</td></tr>
<tr><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td><td>0</td><td>1</td><td>1</td></tr>
<tr><td colspan="8">DELIVER 2 STAMPS, 0 NICKELS, 1 1¢ STAMP:</td></tr>
<tr><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td></tr>
<tr><td colspan="8">COIN RETURN:</td></tr>
<tr><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
</table>

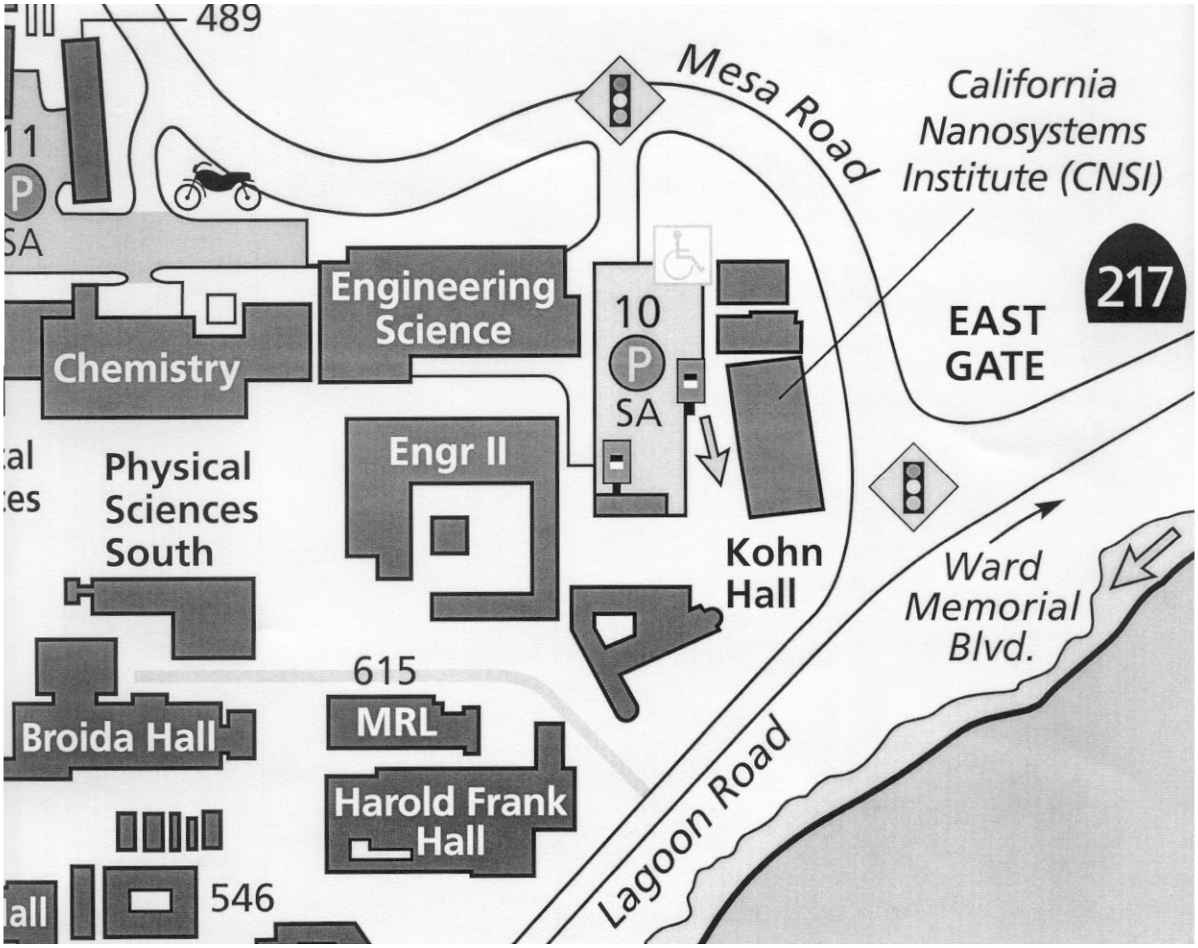
3. A state diagram.



I will grade this problem by applying a variety of input sequences and determining if your design meets the spec.

Problem #2.

In this problem you are to design a controller for the traffic lights at the entrance to Parking structure 10 on Mesa Road. A map of the area is shown below.



There are two sensors in the road, one in the left turn lane of Mesa Road (into the parking structure) and one at the exit of parking structure 10. There are four traffic lights to control: the east signal along Mesa Road (toward the east gate), the west signal along Mesa Road (away from the east gate), the left turn signal into the parking structure and the left/right turn signal out of the parking structure.

Normally, the through traffic lights along Mesa Road (east and west) are green, while the left turn and exit signals are red. When either of the two sensors becomes active, the appropriate signal(s) on Mesa Road should first turn yellow, then red; the signal corresponding to the active sensor should turn green. When the sensor becomes inactive, the appropriate turn signal should turn yellow, then red, along with the appropriate Mesa Road signal turning green.

There are also pedestrian crossing buttons on the south side of Mesa Road, on either side of parking structure 10. When pressed, the lights should respond in a manner which allows pedestrians to cross without being run over.

In all cases, the sequence of traffic light transitions is **green → yellow→ red→ green**. Transitions from red to green can occur simultaneously with transitions from yellow to red (assume a fixed delay from red to green is built into the traffic signal), but direct transitions from green to red and yellow to green are not allowed.

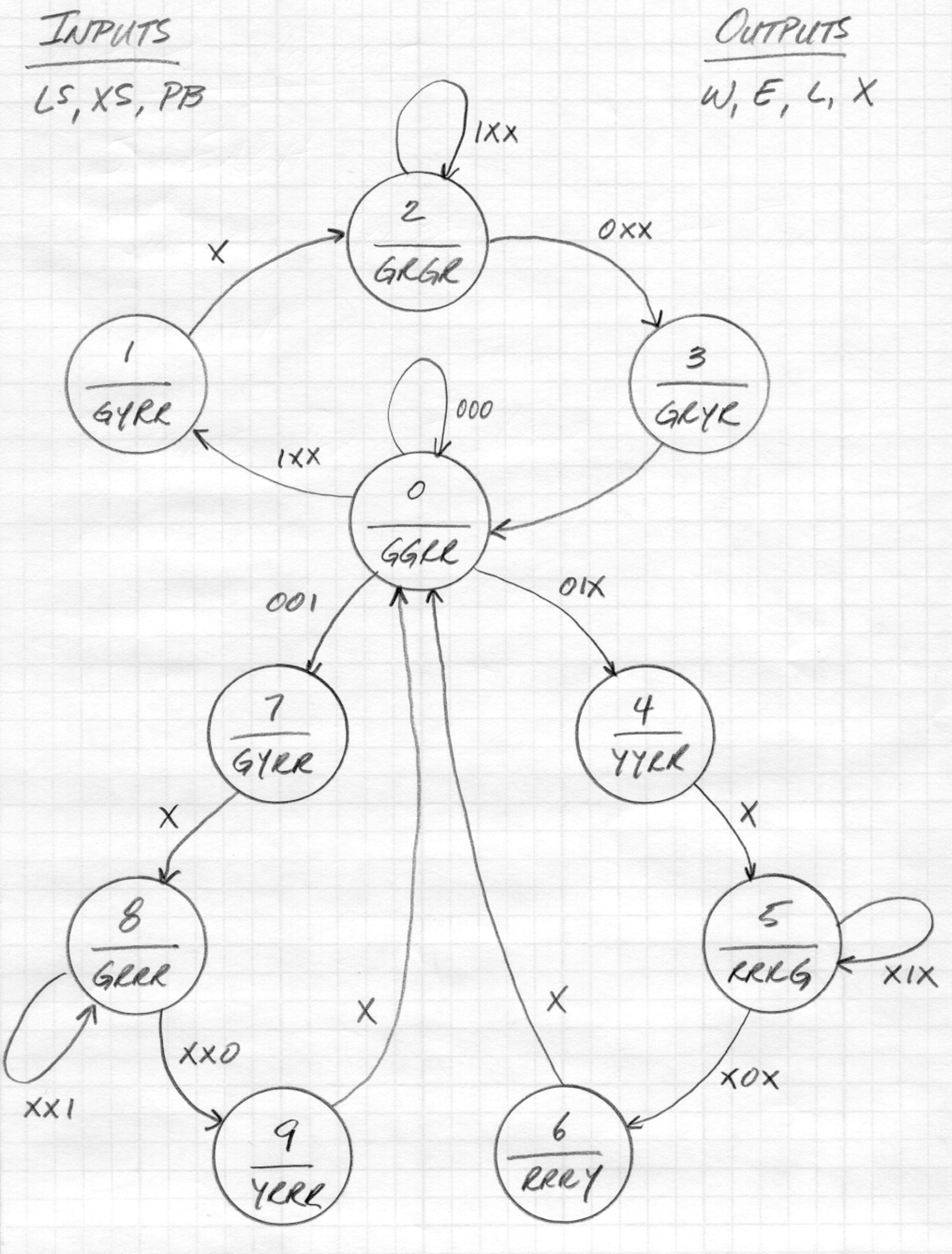
The priority of inputs is (1) left turn signal into the parking structure, (2) exit signal from parking structure and (3) pedestrian crossing. You can assume that any and all inputs can be active at any time.

Assume that each traffic light is driven by a 2-bit output (from the controller) corresponding to the following:

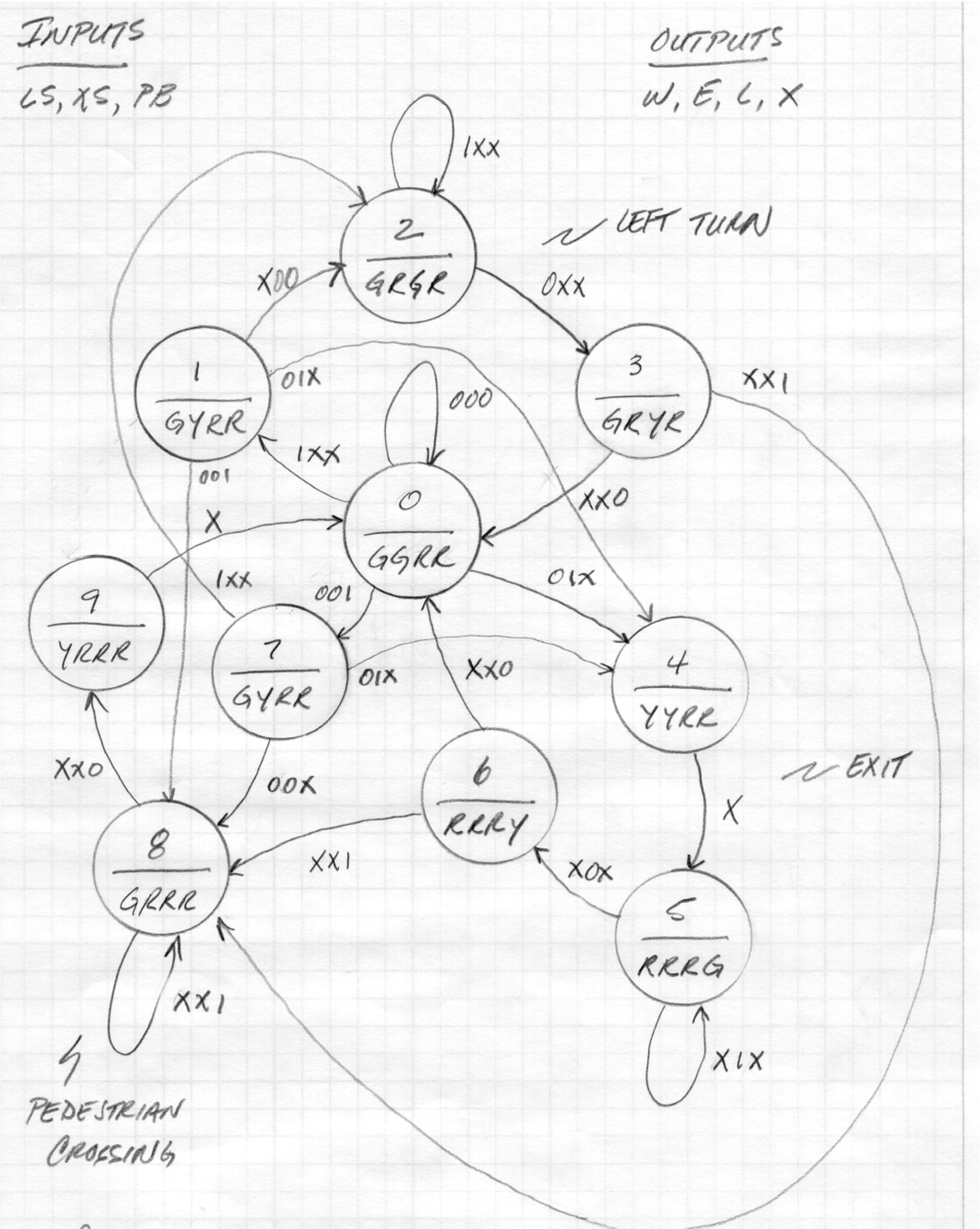
00	green	(G)
01	yellow	(Y)
10	red	(R)
11	not used	

Design the controller as a Moore machine and in such a way that traffic flow is maximized and accidents are minimized. Clearly state any and all assumptions.

You only need to provide a state diagram. On your state diagram, indicate the output to the traffic light by color (single character), not by binary code (it makes it easier to read). Order and label your outputs as follows: MesaWest (W), MesaEast (E), LeftTurn (L), Exit (X). Also, inputs should be labeled left sensor (LS), exit sensor (XS) and pedestrian button (PB) on the state diagram in order of priority. The initial state should be labeled state 0, and have an output of GGRR.



BASIC SOLUTION WITH 3 DIFFERENT SIGNAL CYCLES FROM COMMON INITIAL STATE



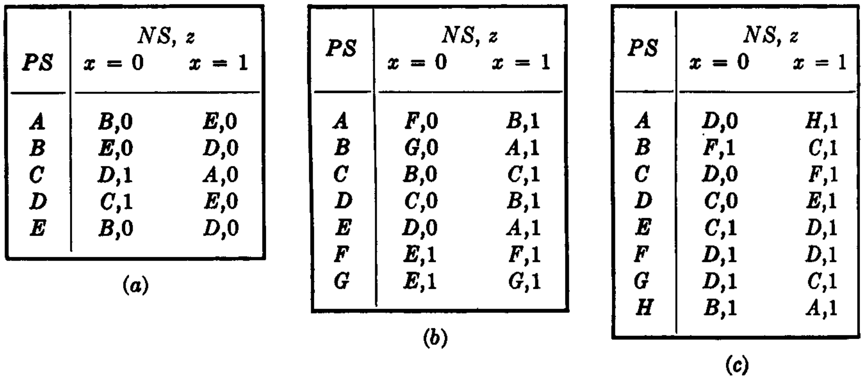
SOLUTION WITH (SOME) MID-CYCLE INTERRUPTS

Problem #3.

For each of the state tables given below:

1. Reduce the state table using an implication chart. In doing the reduction, list which implied pairs are eliminated on each pass.
2. Using the Moore reduction procedure, find the equivalence partition for each of the state tables. Show all steps of the reduction.

Table P10-6



PS	*NS, z*	
	$x = 0$	$x = 1$
A	*B*,0	*E*,0
B	*E*,0	*D*,0
C	*D*,1	*A*,0
D	*C*,1	*E*,0
E	*B*,0	*D*,0

(*a*)

PS	*NS, z*	
	$x = 0$	$x = 1$
A	*F*,0	*B*,1
B	*G*,0	*A*,1
C	*B*,0	*C*,1
D	*C*,0	*B*,1
E	*D*,0	*A*,1
F	*E*,1	*F*,1
G	*E*,1	*G*,1

(*b*)

PS	*NS, z*	
	$x = 0$	$x = 1$
A	*D*,0	*H*,1
B	*F*,1	*C*,1
C	*D*,0	*F*,1
D	*C*,0	*E*,1
E	*C*,1	*D*,1
F	*D*,1	*D*,1
G	*D*,1	*C*,1
H	*B*,1	*A*,1

(*c*)

B	~~B-E D-E~~			
C	X	X		
D	X	X	~~A-E~~	
E	~~D-E~~	✓	X	X
	A	B	C	D

PASS 1: ELIMINATION OF DIFFERING OUTPUTS

(AC) (AD) (BC) (BD) (CE) (DE)

B AND E ARE EQUIVALENT (B-E, D-D)

PASS 2: ELIMINATION OF IMPLIED PAIRS

(AB) (AE) (DE) ⟶ (BE) ONLY EQUIVALENT PAIR

$P_0 = (ABCDE)$

$P_1 = \underset{1}{(ABE)}\underset{2}{(CD)}$

$A_1^1 \quad B_2^1 \quad E_2^1 \qquad C_1^2 \quad D_1^2$

$P_2 = \underset{1}{(A)}\underset{2}{(BE)}\underset{3}{(CD)}$

$B_3^2 \quad E_3^2 \qquad C_1^3 \quad D_3^2$

$$P_3 = \underset{1}{(A)}\underset{2}{(BE)}\underset{3}{(C)}\underset{4}{(D)}$$

$$B_4^2 \quad E_4^2$$

P_3 = EQUIVALENCE PARTITION = $\underline{(A)(BE)(C)(D)}$

	NS, z	
PS	x=0	x=1
A	B,0	B,0
B	B,0	D,0
C	D,1	A,0
D	C,1	B,0

B	F-G A-B					
C	~~B-F~~ ~~B-C~~	~~B-G~~ ~~A-C~~				
D	~~C-F~~	~~C-G~~ ~~A-B~~	~~B-C~~			
E	~~D-F~~ ~~A-B~~	~~D-G~~	~~B-D~~ ~~A-C~~	~~C-D~~ ~~A-B~~		
F	X	X	X	X	X	
G	X	X	X	X	X	✓
	A	B	C	D	E	F

PASS 1

(AF)(AG)(BF)(BG)(CF)(CG)(DF)(DG)(EF)(EG)

(FG) ARE EQUIVALENT

PASS 2

(AC)(AD)(AE)(BC)(BD)(BE)(CD)(CE)

(AB) ARE EQUIVALENT

$\Longrightarrow$ (AB)(C)(D)(E)(FG)

$P_0 = (ABCDEFG)$

$P_1 = (\underset{1}{ABCDE})(\underset{2}{FG})$

$A_1^2 \quad B_1^2 \quad C_1^1 \quad D_1^1 \quad E_1^1$ (FG EQUIVALENT)

$P_2 = (\underset{1}{AB})(\underset{2}{CDE})(\underset{3}{FG})$

$C_2^1 \quad D_1^2 \quad E_1^2$ (AB EQUIVALENT)

$P_3 = (\underset{1}{AB})(\underset{2}{C})(\underset{3}{DE})(\underset{4}{FG})$

$D_1^2 \quad E_A^3$

$P_4 = \underline{(AB)(C)(D)(E)(FG)}$

EQUIVALENCE PARTITION)

PS	x=0	x=1
A	F,0	A,1
C	A,0	C,1
D	C,0	A,1
E	D,0	A,1
F	E,1	F,1

B	X						
C	~~F-H~~	X					
D	~~C-D~~ ~~E-H~~	~~C-F~~ ~~C-E~~	E-F				
E	X	~~C-F~~ ~~C-D~~	X	X			
F	X	~~D-F~~ ~~C-D~~	X	X	C-D		
G	X	~~D-F~~	X	X	C-D	C-D	
H	X	~~B-F~~ ~~A-C~~	X	X	~~B-C~~ ~~A-D~~	~~B-D~~ ~~A-D~~	~~B-D~~ ~~A-C~~
	A	B	C	D	E	F	G

PASS 1:

(AB)(AE)(AF)(AG)(AH)(BC)(CE)(CF)(CG)(CH)
(DE)(DF)(DG)(DH)

PASS 2:

(BD)(BE)(BF)(BG)(BH)(EH)(FH)(GH)

PASS 3:

(AC)(AD)

(CD) (EF)(EG)(FG)

$E \equiv F \equiv G \implies (EFG)$

(A)(B)(CD)(EFG) (H)

$P_0 = (ABCDEFGH)$

$P_1 = \underset{1}{(ACD)}\underset{2}{(BEFGH)}$

$A^{1}_{2} \quad C^{1}_{2} \quad D^{1}_{2}$

$B^{2}_{1} \quad E^{1}_{1} \quad F^{1}_{1} \quad G^{1}_{1} \quad H^{2}_{1}$

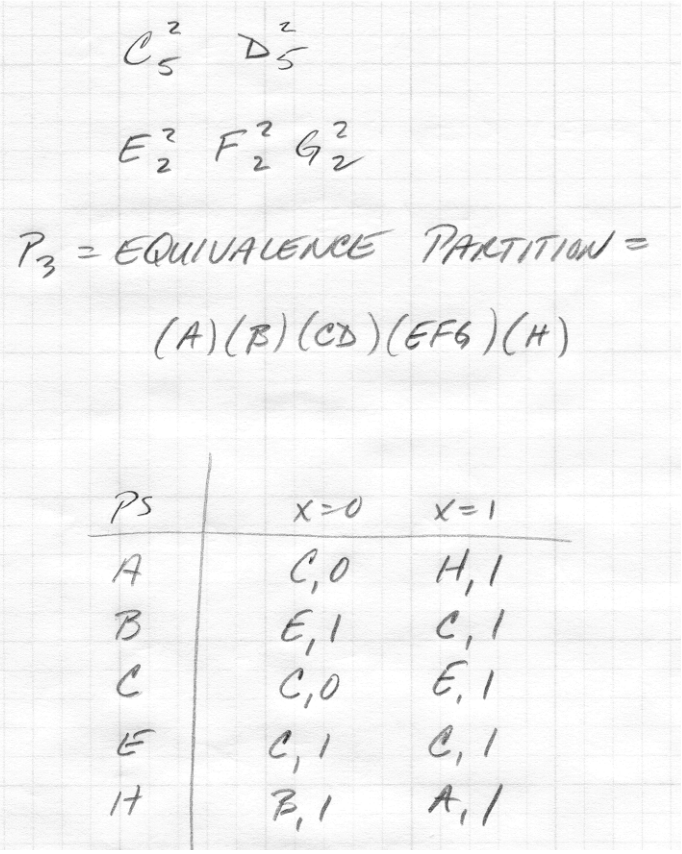
$P_2 = \underset{1}{(ACD)}\underset{2}{(BH)}\underset{3}{(EFG)}$

$A^{1}_{2} \quad C^{1}_{3} \quad D^{1}_{3}$

$B^{3}_{1} \quad H^{2}_{1}$

$E^{1}_{1} \quad F^{1}_{1} \quad G^{1}_{1}$

$P_3 = \underset{1}{(A)}\underset{2}{(CD)}\underset{3}{(B)}\underset{4}{(H)}\underset{5}{(EFG)}$



$C_5^2 \quad D_5^2$

$E_2^2 \quad F_2^2 \quad G_2^2$

P_3 = EQUIVALENCE PARTITION =

(A)(B)(CD)(EFG)(H)

PS	x=0	x=1
A	C,0	H,1
B	E,1	C,1
C	C,0	E,1
E	C,1	C,1
H	B,1	A,1

Problem #4.

Find a reduced state table for each of the incompletely specified state tables given below.

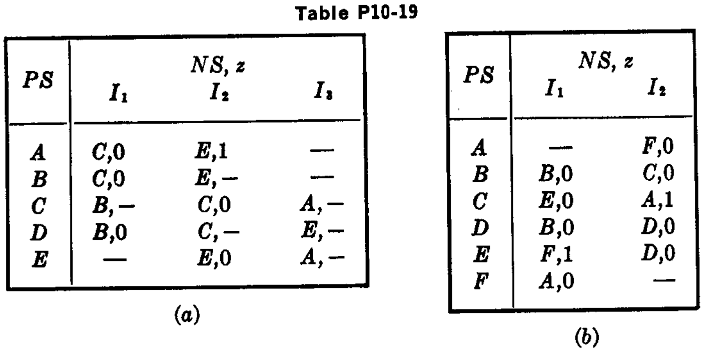


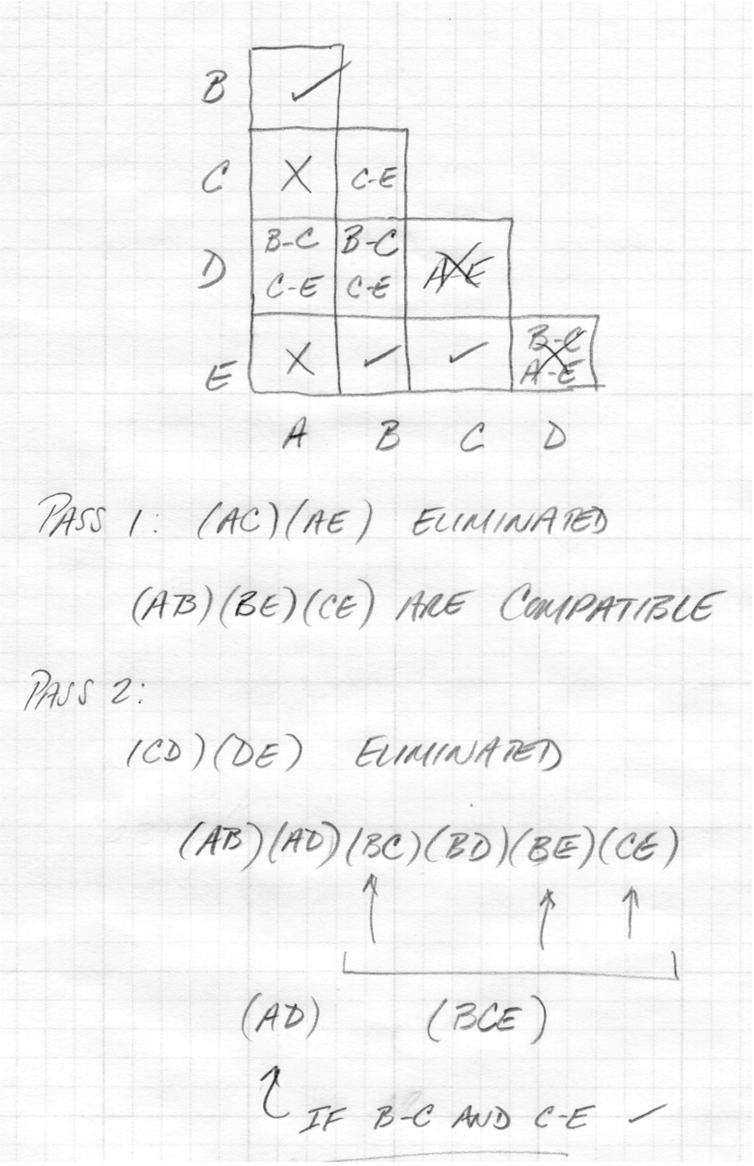
Table P10-19

PS	NS, z I_1	NS, z I_2	NS, z I_3
A	C,0	E,1	—
B	C,0	E,–	—
C	B,–	C,0	A,–
D	B,0	C,–	E,–
E	—	E,0	A,–

(a)

PS	NS, z I_1	NS, z I_2
A	—	F,0
B	B,0	C,0
C	E,0	A,1
D	B,0	D,0
E	F,1	D,0
F	A,0	—

(b)



B	✓			
C	X	C-E		
D	B-C C-E	B-C C-E	~~A-E~~	
E	X	✓	✓	~~B-C~~ ~~A-E~~
	A	B	C	D

PASS 1: (AC)(AE) ELIMINATED

(AB)(BE)(CE) ARE COMPATIBLE

PASS 2:

(CD)(DE) ELIMINATED

(AB)(AD)(BC)(BD)(BE)(CE)

(AD) (BCE)

IF B-C AND C-E ✓

PS	I_1	I_2	I_3
A (D)	B, 0	B, 1	B, –
B (CE)	B, 0	B, 0	A, –

B	C-F				
C	X	X			
D	F-D	~~C-D~~	X		
E	F-D	X	X	X	
F	✓	A-B	A-E	A-B	X
	A	B	C	D	E

PASS 1:

(AC) (BC) (BE) (CD) (CE) (DE) (EF) ELIMINATED

(AF) COMPATIBLE

PASS 2:

(BD) ELIMINATED

PS	x=0	x=1
A= (AF)	A, 0	A, 0
B	B, 0	C, 0
C	E, 0	A, 1
D	B, 0	D, 0
E	A, 1	D, 0

Problem #5.

For the machine defined below:

PS	NS x=0	NS x=1	z
A	D	A	0
B	G	F	1
C	F	C	1
D	H	A	0
E	G	B	0
F	B	C	1
G	E	B	0
H	A	H	0

1. Find a reduced, equivalent machine using an implication chart.
2. Verify that the machine found in part 1 is minimized by determining the equivalence partition.

B	X						
C	X						
D		X	X				
E		X	X				
F	X			X	X		
G		X	X			X	
H		X	X			X	
	A	B	C	D	E	F	G

FIRST PASS

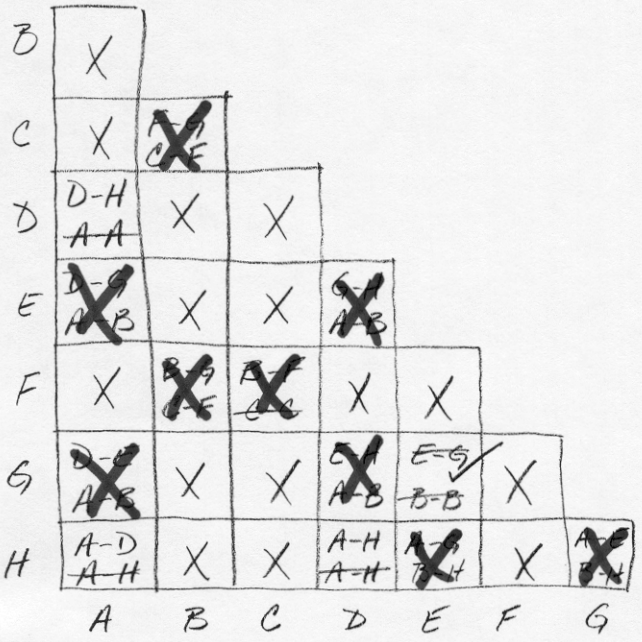
- COMPARE OUTPUTS
- X's WHERE DIFFER

B	X						
C	X	F-G C-F					
D	D-H ~~A-A~~	X	X				
E	D-G A-B	X	X	G-H A-B			
F	X	B-G C-F	B-F ~~C-G~~	X	X		
G	D-E A-B	X	X	E-H A-B	~~E-G~~ ✓ ~~B-B~~	X	
H	A-D ~~A-H~~	X	X	A-H ~~A-H~~	A-G B-H	X	A-E B-H
	A	B	C	D	E	F	G

SECOND PASS

- FIND IMPLIED PAIRS
- REMOVE SELF IMPLIED PAIRS
  (E-G IN BOX EG, ETC.)
- REMOVE SAME STATE PAIRS
  (BB IN BOX EG, ETC.)

⟹ EG ARE EQUIVALENT



	A	B	C	D	E	F	G
B	X						
C	X	~~A-G~~ ~~C-F~~					
D	D-H ~~A-A~~	X	X				
E	~~D-G~~ ~~A-B~~	X	X	~~G-H~~ ~~A-B~~			
F	X	~~B-G~~ ~~C-F~~	~~B-F~~ ~~C-C~~	X	X		
G	~~D-E~~ ~~A-B~~	X	X	~~C-H~~ ~~A-B~~	~~E-G~~ ✓ ~~B-B~~	X	
H	A-D ~~A-H~~	X	X	A-H ~~A-H~~	~~A-G~~ ~~B-H~~	X	~~A-E~~ ~~B-H~~

THIRD PASS

- ELIMINATE IMPLIED PAIRS
- COMBINE EQUIVALENT PAIRS

$\underbrace{(AD)(AH)(DH)}$

(ADH)

- (ADH)(B)(C)(D)(EG)(F)

$$P_0 = (ABCDEFGH)$$

$$P_1 = \underset{1}{(ADEGH)}\underset{2}{(BCF)}$$

$$A^1_1 \quad D^1_1 \quad E^1_2 \quad G^1_2 \quad H^1_1$$

$$B^1_2 \quad C^2_2 \quad F^2_2$$

$$P_2 = \underset{1}{(ADH)}\underset{2}{(EG)}\underset{3}{(B)}\underset{4}{(CF)}$$

$$A^1_1 \quad D^1_1 \quad H^1_1$$

$$E^2_3 \quad G^2_3$$

$$C^4_4 \quad F^3_4$$

$$P_3 = \underset{1}{(ADH)}\underset{2}{(EG)}\underset{3}{(B)}\underset{4}{(C)}\underset{5}{(F)} \checkmark$$

$$A^1_1 \quad D^1_1 \quad H^1_1$$

$$E^3_2 \quad G^3_2$$

PS	NS x=0	NS x=1	z
ADH	ADH	ADH	0
B	EG	F	1
C	F	C	1
EG	EG	B	0
F	B	C	1

Problem #6.

For the state table below:

	NS		z	
PS	x=0	x=1	x=0	x=1
A	B	D	0	1
B	C	E	1	1
C	A	F	0	1
D	G	C	1	1
E	A	F	0	0
F	D	H	1	1
G	F	C	1	1
H	A	G	0	1

1. Find and list all equivalent pairs and states using an implication chart.

B	X						
C		X					
D	X		X				
E	X	X	X	X			
F	X		X		X		
G	X		X		X		
H		X		X	X	X	X
	A	B	C	D	E	F	G

B	X						
C	~~A-B~~ ~~D-F~~	X					
D	X	~~C-G~~ ~~C-E~~	X				
E	X	X	X	X			
F	X	~~C-D~~ ~~E-H~~	X	D-G C-H	X		
G	X	~~C-F~~ ~~C-E~~	X	G-F ~~C-C~~	X	D-F C-H	
H	~~A-B~~ ~~D-G~~	X	~~A-A~~ F-G	X	X	X	X
	A	B	C	D	E	F	G

EQUIVALENT PAIRS

(CH)(DF)(DG)(FG)

EQUIVALENT STATES

(CH) (DFG) ... (A)(B)(E)

2. Verify the solution in part 1 by finding the equivalence partition using the Moore reduction procedure.

$P_0 = (ABCDEFGH)$

$P_1 = \underset{1}{(ACH)}\underset{2}{(BDFG)}\underset{3}{(E)}$

$A^2_2 \; C^1_2 \; H^1_2 \quad B^1_3 \; D^2_1 \; F^2_1 \; G^2_1$

$P_2 = \underset{1}{(A)}\underset{2}{(CH)}\underset{3}{(B)}\underset{4}{(DFG)}\underset{5}{(E)}$

$C^1_4 \; H^1_4 \quad D^4_2 \; F^4_2 \; G^4_2$

EQUIVALENCE PARTITION:

$(A)(B)(CH)(DFG)(E)$

3. Construct the reduced state table

	NS		Z	
PS	X=0	X=1	X=0	X=1
A	B	DFG	0	1
B	CH	E	1	1
CH	A	DFG	0	1
DFG	DFG	CH	1	1
E	A	DFG	0	0

Problem #7.

1. Find the equivalence partition for the state table given below. Show all interim partitions generated in the reduction process and also how the partition was generated (how/why pairs are eliminated).

	NS,z	
PS	x=0	x=1
A	G,0	F,1
B	B,1	C,1
C	B,0	F,1
D	A,1	F,1
E	G,0	F,1
F	D,0	D,1
G	B,1	E,1

$P_0 = (ABCDEFG)$

$P_1 = (\underset{1}{ACEF})(\underset{2}{BDG})$

$A \begin{array}{l} \nearrow G(2) \\ \searrow F(1) \end{array} \quad C \begin{array}{l} \nearrow B(2) \\ \searrow F(1) \end{array} \quad E \begin{array}{l} \nearrow G(2) \\ \searrow F(1) \end{array} \quad F \begin{array}{l} \nearrow D(2) \\ \searrow D(2) \end{array}$

$B \begin{array}{l} \nearrow B(2) \\ \searrow C(1) \end{array} \quad D \begin{array}{l} \nearrow (1) \\ \searrow (1) \end{array} \quad G \begin{array}{l} \nearrow B(2) \\ \searrow E(1) \end{array}$

$P_2 = (\underset{1}{ACE})(\underset{2}{F})(\underset{3}{BG})(\underset{4}{D})$

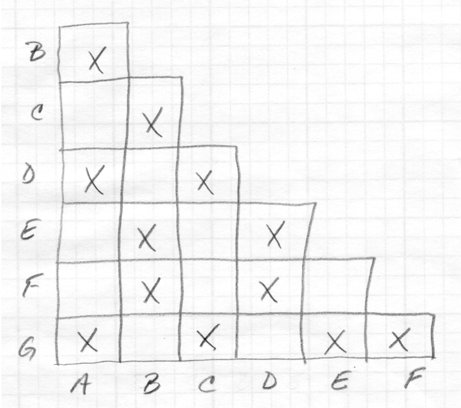
$A \begin{array}{l} \nearrow G(3) \\ \searrow F(2) \end{array} \quad C \begin{array}{l} \nearrow B(3) \\ \searrow F(2) \end{array} \quad E \begin{array}{l} \nearrow G(3) \\ \searrow F(2) \end{array}$

$B \begin{array}{l} \nearrow B(3) \\ \searrow C(1) \end{array} \quad G \begin{array}{l} \nearrow B(3) \\ \searrow E(1) \end{array}$

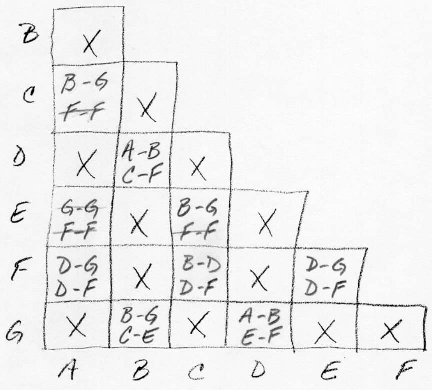
$P_3 = (ACE)(F)(BG)(D) = P_2$

$\Longrightarrow$ EQUIVALENCE PARTITION

2. Construct an implication chart and verify the result of part 1. You only need to draw one implication chart, but list (1) the pairs eliminated and (2) the pairs found to be equivalent on each pass.



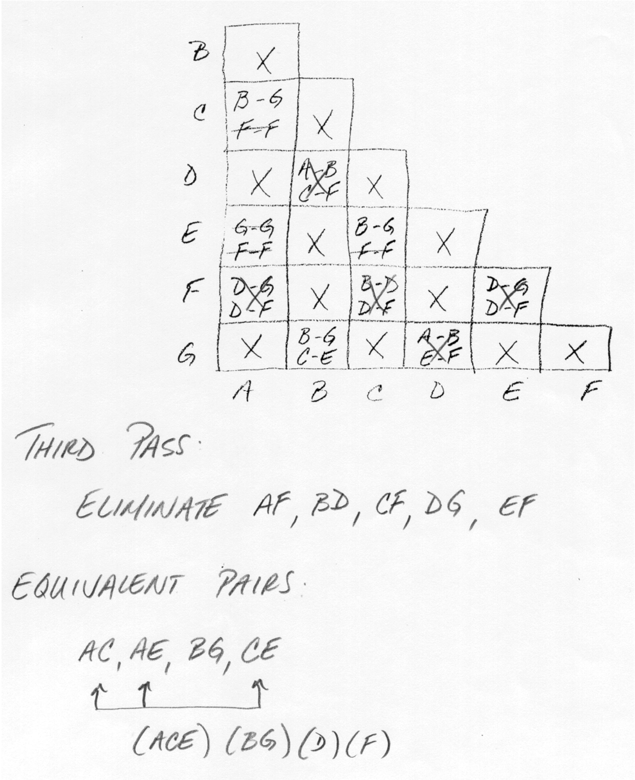
	A	B	C	D	E	F
B	X					
C		X				
D	X		X			
E		X		X		
F		X		X		
G	X		X		X	X



	A	B	C	D	E	F
B	X					
C	B-G ~~F-F~~	X				
D	X	A-B C-F	X			
E	~~G-G~~ ~~F-F~~	X	B-G ~~F-F~~	X		
F	D-G D-F	X	B-D D-F	X	D-G D-F	
G	X	B-G C-E	X	A-B E-F	X	X

- ELIMINATE SAME STATE PAIRS IN CELLS AC, AE, CE

- ROWS A AND E ARE IDENTICAL (NO IMPLIED PAIRS) SO THEY ARE THE SAME STATE



B	X					
C	B-G ~~F-F~~	X				
D	X	~~A-B~~ ~~C-F~~	X			
E	~~G-G~~ ~~F-F~~	X	B-G ~~F-F~~	X		
F	~~D-G~~ ~~D-F~~	X	~~B-D~~ ~~D-F~~	X	~~D-G~~ ~~D-F~~	
G	X	B-G C-E	X	~~A-B~~ ~~E-F~~	X	X
	A	B	C	D	E	F

THIRD PASS:

ELIMINATE AF, BD, CF, DG, EF

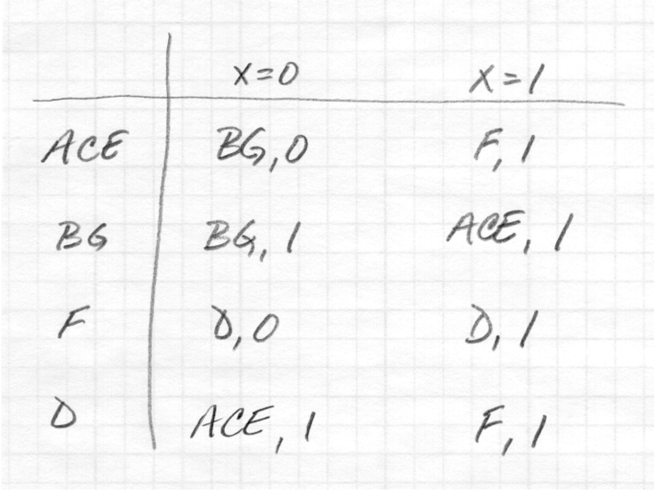
EQUIVALENT PAIRS:

AC, AE, BG, CE

↑ ↑ ↑

(ACE) (BG) (D) (F)

3. Construct the reduced state table.



	X=0	X=1
ACE	BG,0	F,1
BG	BG,1	ACE,1
F	D,0	D,1
D	ACE,1	F,1